Long Swings with Memory and Stock Market Fluctuations

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Abstract Since the seminal papers of LeRoy and Porter (1981) and Shiller (1981), the debate on the issue of whether the stock market "over-reacts" has still not been settled for financial economists. In this paper, we show that long swings with memory in the dividend process — a property which has been neglected in the literature — could cause the stock price to move as if a bubble exists, which results in excess volatility. The efficient method of moments (EMM) procedure is used to examine the long swing property in the dividend series. We then show by simulation that excess volatility could be generated even in large samples.

1 Introduction

The pioneering research by LeRoy and Porter (1981) and Shiller (1981) marks the beginning of a debate on whether the Efficient Market Hypothesis (EMH) can be reconciled with observed price and dividend volatilities.[1] Under the conventional EMH, because a theoretically warranted (*ex ante*) variable is the best predictor of the observed (*ex post*) variable, the variance of the *perfect foresight* price or dividend-price ratio should be larger than the variance of the observed price or dividend-price ratio.[2] Contrary to what it should have been, the volatility tests in early research seem to indicate that stock price fluctuations are too large to result solely from changes in the present discounted value of expected dividends. This has been interpreted as implying drawbacks of the theoretical models if not the EMH.

In this paper, we will examine the extent to which variations in stock prices can be related to long swings in the dividends. A casual observation would indicate that a large part of fluctuations in stock prices appear to be related to such long swings. We model the long swings in dividends by allowing for duration dependence in the duration structure of a particular swing. This duration dependence will introduce *timing activities* in the part of economic agents. Investors will anticipate the end of each swing and thus additional dynamics could be generated in the stock prices. Our empirical evidence supports the argument that there are long swings with negative duration dependence in the dividend series. Using the newly proposed Efficient Method of Moments estimation, we find that long swings with duration dependence appear to be a key characteristic of the dividend series.

2 Modelling Dividend and Stock Price Processes

In this section, we will show how the conventional present value model can be used to price a continuous stream of dividends with potential long swings and duration dependence in the swings. While no simple closed form solution for the stock price can be found without further assumptions, we provide general formulas on which we can evaluate the model numerically.

As in Campbell and Kyle (1993) and others, we model the process of dividends in a continuous-time framework. Assume that the long term cost of equity is constant. Then the stock price is modelled as the discounted value of expected future dividends given by

$$P_t^* = E_t(\int_t^\infty e^{-r(u-t)} D_u du), \qquad (1)$$

where P_t^* is the current stock price at time t, r is the continuously compounded risk adjusted discount rate (cost of capital) applied to random rate of dividends D_u received at time u, and $E_t(\cdot) \equiv E(\cdot \mid \mathcal{F}_t)$ is the expectation operator at time t conditional on the information set $\mathcal{F}_t$. The informa-

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[1] See LeRoy (1989) for a survey.

[2] The *perfect foresight* series is the series that would prevail if investors have perfect knowledge of the relevant variables, such as the future dividends in the *ex ante* stock price identity (1) of Section 2 below.

tion set $\mathcal{F}_t$ is usually assumed to consist of *only* individual dividend series $\{D_u : u \leq t\}$, so the variation in P_t^* is basically due to the time variation in the $\{D_t\}$ series.

In the following, we argue that in addition to the $\mathcal{F}_t$ as conventionally assumed, economic agents will form expectations conditional on an "enlarged" information set $\mathcal{H}_t$ which contains $\mathcal{F}_t$ and other information about the dividend process. Specifically, we suggest that the growth rates g_t of dividends $\{D_t\}$ are governed by a state vector with different regimes and follow a semi-Markov model:

$$\begin{aligned} d\log D_t &= g_t\,dt, \\ g_t &= s_t + h_t, \end{aligned} \tag{2}$$

where s_t represents the mean level of g_t in a given state or regime, and h_t is a component reflecting short term fluctuations in g_t. The duration of g_t in any particular regime, τ, is assumed to be distributed according to certain distribution. Such a model can be motivated from changes in the dividend policies or from developments in the economy that could affect the behavior of individual firms.

To keep the presentation simple and intuitive, we will consider the case of only two regimes in the dividend growth rates: expansion and contraction with $s_t = \overline{s}$ and $\underline{s}$, respectively. The short term fluctuation in g_t is assumed to follow a continuous-time AR(1) or Ornstein–Uhlenbeck process:

$$dh_t = ah_t dt + \sigma dW_t, \tag{3}$$

where dW_t is a zero-mean, unit variance (i.e., standard) Brownian motion, σ is the standard deviation of innovations in g_t, and the parameter a measures the speed of mean reversion.

Let τ_b be the backward stopping time (i.e., time into a regime) and τ_f be the forward stopping time (i.e., $\tau_f = \tau - \tau_b$). Assume that economic agents know the structure of the data generating process for dividends. Since both the current dividend growth rate g_t and the magnitude of the swings s_t can be observed, a rational investor will include these information in forming expectations, i.e., $\{g_t, s_t, h_t, \tau_b\} \subset \mathcal{H}_t$. Then the expected *level* of dividends at some future point in time conditional on the $\mathcal{H}_t$ is given by

$$\begin{aligned} E(D_u \mid \mathcal{H}_t) &= E_t(\exp(s_t(u-t) + \textstyle\int_t^u h_v dv D_t)) \\ &= M(s_t, h_t, u-t)D_t, \end{aligned} \tag{4}$$

where

$$\begin{aligned} &M(s_t, h_t, u-t) = \\ &\exp(s_t(u-t) + \tfrac{h_t}{a}(e^{a(u-t)} - 1) + \\ &\sigma^2(\tfrac{e^{2a(u-t)}}{4a^3} - \tfrac{e^{a(u-t)}}{a^3} + \tfrac{u-t}{2a^2} + \tfrac{3}{4a^3})). \end{aligned} \tag{5}$$

From the present value model (1) and equation (4), we can "conjecture" that the discounted expected price at the end of a regime is equal to a non-linear function of the current dividend level. Specifically, we define

$$\begin{aligned} A(s, h_t)D_t \;\equiv\; & E_{s_t, \tau_b = \tau}(\lim_{\Delta t \to 0^+}(e^{-r\Delta t} P_{t+\Delta t} + \\ & \textstyle\int_t^{t+\Delta t} e^{-r(u-t)} D_u du)). \end{aligned} \tag{6}$$

The time subscript for s_t can be omitted in $A(\cdot)$, since the following discussions will show that $A(\cdot)$ depends on the regime of g_t and not the corresponding mean level. If $\{D_t\}$ follows the semi-Markov process specified in equations (2) and (3), then with equation (4) the *ex ante* stock price would be equal to

$$P_t = E(P_t^* \mid \mathcal{H}_t) = B(s, h_t, \tau_b)D_t \tag{7}$$

where

$$\begin{aligned} &B(s, h_t, \tau_b) = \\ &E_{s,\tau_b}(e^{-r\tau_f} M(s, h_t, \tau_f) E_{h_t}(A(s, h_{t+\tau_f})) + \\ &\textstyle\int_t^{t+\tau_f} e^{-r(u-t)} M(s, h_t, u) du). \end{aligned} \tag{8}$$

Note that $B(s, h_t, \tau_b)$ can also be expressed as

$$B(s, h_t, \tau_b) = \exp(-\delta(s, h_t, \tau_b))$$

where $\delta_t \equiv \ln(D_t/P_t)$ is the log dividend-price ratio. Given the form of $B(s, h_t, \tau_b)$ in (8), it should be clear from the definition in (6) that

$$\begin{aligned} A(s, h_t) &= E_s(M(s', h_t, 0)\, B(s', h_t, 0)) \\ &= E_s(B(s', h_t, 0)), \end{aligned} \tag{9}$$

where s' is a regime for g_t different from s. Substituting equation (8) into equation (9), we will have a functional equation:

$$A(\cdot) = F(A(\cdot)),$$

with

$$\begin{aligned} F(\cdot) \;=\; & E_s(E_{s',\tau}(e^{-r\tau_f} M(s', h_t, \tau) E_{h_t}(\cdot) + \\ & \textstyle\int_t^{t+\tau} e^{-r(u-t)} M(s', h_t, u) du)) \end{aligned}$$

and $A(s, h_t)$ is just the fixed point of this non-trivial functional equation. While no closed form formula for $A(s, h_t)$ can be found without more assumptions, we can solve the above functional equation numerically, thereby giving the stock price conditioning on the information set $\mathcal{H}_t$.

3 Timing Activity and Stock Market Fluctuations

The key driving force for explaining the phenomenon of "excess volatility" in the above model can be seen from the stock price equation (7). This equation has a major difference from the traditional approach in that the backward stopping time τ_b enters the equation through $B(s, h_t, \tau_b)$ in equation (8). With the knowledge that the economy has been in a certain regime for a length of time τ_b, a rational economic agent will form an expectation on how much longer the economy will stay in the current regime (i.e., the forward stopping time τ_f). This is the *timing activity* of economic agents which is very common in reality. While this timing activity would be time invariant for a Markov process or a memoryless process, it is not so in a world when there is duration dependence. In particular, excess volatility will "result" when there is negative duration dependence in the regime spells as discussed below.[3]

To show that the timing activity creates "excessive dynamics" in the stock market, Figure 1 shows a diagram for the log dividend-price ratio δ_t according to the parameters $\overline{s} = 0.0191$ in the expansion/upswing regime and $\underline{s} = -0.0733$ in the contraction/downswing regime, and the standard deviation of the h_t component is 0.1145.[4] To see more carefully why the timing activity will generate more volatility, we can look at the case when τ_b increases from 2 to 3 (years) in the $\overline{s}$ regime, and h_t from -0.1534 to 0.1534. The contribution to the log dividend-price ratio, then, for each individual part would be 0.1153 (s_t) and 0.0893 (h_t). The contribution of the timing of the regime contribute significantly ($0.1015/0.03663 = 2.7710$) if compared with the contribution of the dynamics out of the short term dynamics ($0.1153/0.0893 = 1.2912$).

As one could see, the additional non-linear variation produced by the timing activity could generate excess volatility. Since the excess volatility has been understood in the literature more or less related to the so-called (rational) "bubbles", such non-linear variation would look very much like a bubble for a researcher in a Markov world, especially when there exists negative duration dependence in the duration probability structure. As the current regime continues, we can see from $B(s, h_t, \tau_b)$ in equation (8) that there is an extra increment which is not due to the fundamentals in the normal sense. In terms of our model, apart from the short term dynamics component h_t, when the expansion (contraction) regime lasts, δ_t decreases (increases) as P_t increases (decreases) more and δ_t^* increases (decreases) as P_t^* decreases (increases) more. That is, using δ_t as a forecast of δ_t^*, the forecast error gets larger as a regime continues.

Therefore, duration dependence in the stock price could introduce non-linearity in a way that as if excess volatility and bubbles show up in the data. That is, if we do not take into account of the duration dependence in the regime and the timing activities arising thereof, then

1. a negative correlation between the forecast and the forecast error will be generated which "results" in excess volatility, and

2. a spurious upward bias exists in the stock price or dividend-price ratio, which will appear as if there exists a bubble and the bubble bursts when a regime ends.

4 Econometric Methods and Empirical Results

In this section, we will examine the potential long swing characteristics of the dividends series empirically, and explore the implications of these characteristics on the stock price behavior. Since the proposed dividend model is in a continuous-time framework with non-trivial regime switching, the estimation could be rather difficult. We thus employ the newly proposed Efficient Method of Moments (EMM) to estimate and test the specification of our model. It should be emphasized that the econometric methods can take account of the fact that the price data are point-sampled, while the dividend data are time-averaged.

4.1 Data on Dividends and Prices

The data set used in this paper is taken from the Center for Research in Security Prices (CRSP) series of monthly returns on the value-weighted New York Stock Exchange (NYSE) index from 1926 to 1994. The CRSP data incorporate careful corrections for stock splits, non-cash distributions, mergers, delisting and other potential problems. Returns are reported both inclusive and exclusive of

[3] Negative duration dependence means the hazard rate goes down the longer we stay in the current regime, or, the more likely we will stay in the current regime (i.e., the larger the expected regime duration). If we have negative duration dependence for the whole domain of backward stopping time, this would mean that the longer we have stayed at the current regime, the longer we will stay at the current regime.

[4] These parameters are chosen based on results in Table 3. The kinks in the curves are due to the special feature of the Pareto distribution as discussed below. It should be emphasized that such a feature is not crucial in our analysis.

dividends. This makes it possible to compute the levels of dividends and prices up to an arbitrary scale factor.

Although the raw CRSP data are available monthly, there is quite a strong seasonality in the monthly dividend series. The annual series will not suffer from seasonality, but will lose much of the dynamics in the data. Therefore, quarterly data are used for empirical analysis here. In aggregating the data to a quarterly interval, we assumed that dividends paid each month are accumulated through the quarter without receiving interest. The quarterly dividend is then the sum of monthly dividend payments, while the quarterly price is formed as the previous quarter's price times the quarterly return excluding dividends, compounded monthly.

Let RD_t and R_t be the value-weighted nominal returns with and without dividends. If we denote the nominal stock price and dividend series as PN_t and DN_t, then $RD_t = (PN_t + DN_t - PN_{t-1})/PN_{t-1}$ and $R_t = (PN_t - PN_{t-1})/PN_{t-1}$. Thus a normalized value-weighted price series, PN_t, is generated by $PN_t = (1 + R_t)PN_{t-1}$ with the price in 1925:IV set equal to one. Then a normalized nominal dividend series, DN_t, is generated by $DN_t = (RD_t - R_t)PN_{t-1}$. Goods price level, PG_t, was constructed from the CPI inflation, π_t, obtained from Ibbotson Associates, by computing $PG_t = (1 + \pi_t)PG_{t-1}$ with the 1925:IV CPI level set equal to one. Once nominal series and goods price series are constructed, real stock price (P_t) and dividend (D_t) series are generated by dividing the nominal values by the corresponding CPI. We shall denote the logs of P_t and D_t by p_t and d_t, respectively.

Table 1 provides support for the motivation of our model. As mentioned before, the excess volatility puzzle based on variance comparison relies heavily on the "fact" that the observed dividend-price ratio δ_t and the prediction error $\varepsilon_t = \delta_t^* - \delta_t$ are uncorrelated in a *linear* space. However, as argued in Section 3, it is possible that δ_t and ε_t are truly uncorrelated in a *nonlinear* framework, but appear to be linearly correlated when there is duration dependence. Table 1 presents a simple diagnostic check of the data, and shows that the "rational forecasts" (δ_t) and the prediction errors (ε_t) are significantly negatively correlated. This suggests that the framework for the conventional excess volatility tests may be of suspect.

$$\varepsilon_t = \theta_0 + \theta_1 \delta_t + e_t$$

	$\hat{\theta}_0$	$\hat{\theta}_1$
Sample	-3.8497	-0.8535
	(0.2152)	(0.0469)
Simulation	-3.7912	-0.8818
(size 25,000)	(0.0245)	(0.0044)

Standard errors are reported in the parentheses.

Table 1: Regression of forecast error on the fundamentals (see Section 4.3 for the simulation)

4.2 Efficient Method of Moments

The idea of using the scores of an auxiliary model (called *score generator*) to summarize systematically the characteristics of the data and confronting a structural model with this score generator is proposed in Bansal, Gallant, Hussey and Tauchen (1995) and Gallant and Tauchen (1996). The estimation method has the desirable feature that if the score generator nests the structural model, the estimator is as efficient as the maximum likelihood estimator. Furthermore, the data dependent expansion of the semi-nonparametric score generator guarantees the score generator would closely approximate the actual distribution of data and the estimator is nearly fully efficient under very general conditions (see Gallant and Long (1996)). The procedure is thus referred to as the Efficient Method of Moments (EMM) estimation. Details on the EMM estimators and their statistical properties can be found in the above papers.

4.3 Results and Implications

Table 2 presents the results of fitting various auxiliary models. Based on the Bayesian information criterion (BIC), we have chosen the semi-nonparametric (SNP) model with $L_\mu = 4$, $L_r = 4$, $L_p = 1$, $K_z = 4$ as the auxiliary model. This model contains 5 parameters which characterize five dimensions of the first moment of the dividend growth rate series, 5 parameters which characterize five dimensions of the second moment of the dividend growth rate series, and 4 parameters which characterize four dimension of the higher moment of the dividend growth rate series. The value of scores corresponding to these parameters could be used to detect the dimension where our structural model fails when normalized in the form of t-ratio.

With the chosen auxiliary model, we estimate the model developed in Section 2 using the Pareto distribution as the duration distribution on (k_1, ∞) with survivor function $(t/k_1)^{-\alpha_1}$ in the contraction regime, and the duration distribution on (k_2, ∞) with survivor function $(t/k_2)^{-\alpha_2}$ in the

L_μ	L_r	L_p	K_z	K_x	l_θ	Obj	BIC
4	4	1	4	0	14	1.1382	1.2811
4	5	1	4	0	15	1.1555	1.3087
4	3	1	4	0	13	1.1499	1.2827
4	4	1	5	0	17	1.1419	1.2951
4	4	1	3	0	13	1.2045	1.3373
4	0	1	0	0	6	1.3349	1.3962
3	0	1	0	0	5	1.3707	1.4218
5	0	1	0	0	7	1.3336	1.4050

Table 2: Fitting of the auxiliary models

expansion regime. The Pareto distribution is chosen because its simple algebraic form and the corresponding numerical advantage compared to other distributions. Specifically, we have intentionally chosen a less restrictive model that allows asymmetric regime duration behavior.

Table 3 presents the results from the EMM estimation on our model. From the table, we can see that our model is not rejected at 10% level. In contrast, the model without regime switching and the model with memoryless switching, are rejected at 10% level. We can perform tests similar to likelihood ratio tests to check for model specifications. For example, to test for the null hypothesis that the regime switching has no memory, we can use the EMM criterion of the exponential duration structure model minus the EMM criterion of the Pareto duration structure model. In this case, we have a statistic equal to $13.835 - 8.826 = 5.009$ with $8 - 6 = 2$ degrees of freedom, and we can reject the null at 10% significance level. Another null hypothesis of interest would be the hypothesis that there is no regime switching. We then have the EMM criterion difference equal to $19.525 - 8.826 = 10.699$ with $11 - 6 = 5$ degrees of freedom, and again we can reject the hypothesis at 10%.

Based on the estimated parameters, we conduct Monte Carlo experiments to assess the excess volatility puzzle. Table 4 presents the distributions of simulated variance ratios based on 300 trials of different sample sizes. It can be seen from our experiments that regardless the length of the simulated sample, the excess volatility of the observed δ_t over the warranted δ_t^* is a "rule" rather than an exception, and it holds even as the sample size grows.

A. Model with regime switching and regime has a memory with Pareto duration distribution
EMM Criterion = 8.826, Dimension = 6, p-value = 0.184

Parameter	Estimate	Confidence Interval
$\underline{s}$	-0.0733	(-0.1945, -0.0035)
k_1	0.0054	(0.0010, 4.5004)
α_1	2.3967	(1.2300, 9.3000)
$\overline{s}$	0.0191	(0.0105, 0.0520)
k_2	1.2131	(0.0600, 2.8034)
α_2	1.2336	(0.8033, 2.8945)
a	-3.3335	(-4.0405, -2.5543)
σ^2	0.0880	(0.0546, 0.1455)

B. Model without regime switching
EMM Criterion = 19.525, Dimension = 11, p-value = 0.053

Parameter	Estimate	Confidence Interval
$\underline{s} = \overline{s}$	1.6830	(1.0445, 2.0935)
a	-4.4181	(-4.9405, -4.0543)
σ^2	0.0657	(0.0446, 0.0955)

C. Model with memoryless regime switching
EMM Criterion = 13.835, Dimension = 8, p-value = 0.0862

Parameter	Estimate	Confidence Interval
$\underline{s}$	-0.0782	(-0.0920, -0.0435)
λ_1	2.0810	(1.0100, 4.5004)
$\overline{s}$	0.0192	(0.0080, 0.0240)
λ_2	0.6669	(0.2001, 1.8034)
a	-3.3524	(-4.0305, -2.5843)
σ^2	0.0839	(0.0542, 0.1459)

Table 3: EMM estimation results

	Sample Size		
Percentile	50 years	100 years	200 years
5%	0.1183	0.1597	0.1797
10%	0.2440	0.2227	0.2551
25%	0.5110	0.4813	0.4340
50%	0.9111	0.7078	0.7443
75%	1.7870	1.3042	2.0764
90%	3.5542	3.9852	2.4889
95%	6.6829	5.0449	3.0160
mean	1.9304	1.6030	1.5207

Table 4: Distribution of the simulated variance ratios

5 Concluding Remarks

In this paper, using the traditional present value model and the rational expectations framework, we are able to replicate the observed "excess volatility" (even in a very large sample) as documented in the literature. Our starting point was to model long swings in the dividends series with duration dependence — a property which has been somewhat neglected in the literature — and proceed to investigate it empirically. This approach is shown to be able to yield significant implications. Long swing will lead to the timing activities of rational economic agents, which introduce extra (non-linear) dynamics into the stock prices. The empirical results indicate that there is significant duration dependence in the dividend growth series, and it could indeed give rise to the phenomenon of "excess volatility" to the same magnitude as reported in the literature. Therefore, when the timing activities of economic agents are taken into account, the "puzzles" of excess volatility and rational bubble in the stock market are really not that puzzling any more.

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